

Heat conduction in a circle

This text is integrating part of the homonymous link in [PEEI: a computer program for the numerical solution of systems of partial differential equations](#).

Coordinate system: polar

System of measurement: International System of Units

Coordinates of polar system: $\{\rho, \alpha\}$ of which $[\rho] \equiv [\text{length}]$ $[\alpha] \equiv [\text{plane angle}]$ $\Re(\rho) \equiv [0, \infty)$
 $\Re(\alpha) \equiv [0, 2 \cdot \pi)$

Unknown functions: $\tau(\rho, \alpha)$ of which $[\tau] \equiv [\text{temperature}]$

Differential analytic model: $\rho \cdot (\partial^2 \tau(\rho, \alpha) / \partial \rho^2) + \partial \tau(\rho, \alpha) / \partial \rho + \rho \cdot Q / \kappa = 0$ of which $Q = 100$ $\kappa = 20$.

Definition set: $\{\{\rho, \alpha\} / \rho \leq \rho_2\}$ $\rho_2 = 10$.

Case 1-1:

Conditions: $\tau(0, \alpha) = 1000$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $\tau(\rho, \alpha) \equiv \tau(0, \alpha) - 0.25 \cdot Q \cdot \rho^2 / \kappa$

Related files: [mad-A.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-1-1.txt](#), [PEEI-sol-1-1.txt](#), [plot-1-1.jpg](#)

Case 1-2:

Conditions: $\tau(0, \alpha) = 1000$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $\tau(\rho, \alpha) \equiv \tau(0, \alpha) - 0.25 \cdot Q \cdot \rho^2 / \kappa$

Related files: [mad-A.txt](#), [points-2.txt](#), PEEI-mem-2.bin, [cond-1-2.txt](#), [PEEI-sol-1-2.txt](#), [plot-1-2.jpg](#)

Case 1-3:

Conditions: $\tau(0, \alpha) = 1000$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $\tau(\rho, \alpha) \equiv \tau(0, \alpha) - 0.25 \cdot Q \cdot \rho^2 / \kappa$

Related files: [mad-A.txt](#), [points-3.txt](#), PEEI-mem-3.bin, [cond-1-3.txt](#), [PEEI-sol-1-3.txt](#), [plot-1-3.jpg](#)

Case 1-4:

Conditions: $\tau(0, \alpha) = 1000$ $\partial \tau(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $\tau(\rho, \alpha) \equiv \tau(0, \alpha) - 0.25 \cdot Q \cdot \rho^2 / \kappa$

Related files: [mad-A.txt](#), [points-4.txt](#), PEEI-mem-4.bin, [cond-1-4.txt](#), [PEEI-sol-1-4.txt](#), [plot-1-4.jpg](#)

Case 1-5:

Conditions: $T(0, \alpha) = 1000$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv T(0, \alpha) - 0.25 \cdot Q \cdot \rho^2 / K$

Related files: [mad-A.txt](#), [points-5.txt](#), PEEI-mem-5.bin, [cond-1-5.txt](#), [PEEI-sol-1-5.txt](#), [plot-1-5.jpg](#)

Case 2-1:

Conditions: $T(\rho_2, \alpha) = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-A.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-2-1.txt](#), [PEEI-sol-2-1.txt](#), [plot-2-1.jpg](#)

Case 2-2:

Conditions: $T(\rho_2, \alpha) = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-A.txt](#), [points-2.txt](#), PEEI-mem-2.bin, [cond-2-2.txt](#), [PEEI-sol-2-2.txt](#), [plot-2-2.jpg](#)

Case 2-3:

Conditions: $T(\rho_2, \alpha) = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-A.txt](#), [points-3.txt](#), PEEI-mem-3.bin, [cond-2-3.txt](#), [PEEI-sol-2-3.txt](#), [plot-2-3.jpg](#)

Case 2-4:

Conditions: $T(\rho_2, \alpha) = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-A.txt](#), [points-4.txt](#), PEEI-mem-4.bin, [cond-2-4.txt](#), [PEEI-sol-2-4.txt](#), [plot-2-4.jpg](#)

Case 2-5:

Conditions: $T(\rho_2, \alpha) = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-A.txt](#), [points-5.txt](#), PEEI-mem-5.bin, [cond-2-5.txt](#), [PEEI-sol-2-5.txt](#), [plot-2-5.jpg](#)

Case 3-1:

Conditions: $K \cdot (\partial T(\rho_2, \alpha) / \partial \rho) + H \cdot (T(\rho_2, \alpha) - T_\infty) = 0$ $H = 50$ $T_\infty = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.5 \cdot \rho_2 \cdot Q / H + 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-B.txt](#), [points-1.txt](#), PEEI-mem-1.bin, [cond-3-1.txt](#), [PEEI-sol-3-1.txt](#), [plot-3-1.jpg](#)

Case 3-2:

Conditions: $K \cdot (\partial T(\rho_2, \alpha) / \partial \rho) + H \cdot (T(\rho_2, \alpha) - T_\infty) = 0$ $H = 50$ $T_\infty = 0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

Solution in [1]: $T(\rho, \alpha) \equiv 0.5 \cdot \rho_2 \cdot Q / H + 0.25 \cdot Q \cdot (\rho_2^2 - \rho^2) / K$

Related files: [mad-B.txt](#), [points-2.txt](#), PEEI-mem-2.bin, [cond-3-2.txt](#), [PEEI-sol-3-2.txt](#), [plot-3-2.jpg](#)

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Related files: [mad-B.txt](#), [points-3.txt](#), PEEI-mem-3.bin, [cond-3-3.txt](#), [PEEI-sol-3-3.txt](#), [plot-3-3.jpg](#)

Case 3-4:

Conditions: $K \cdot (\partial T(\rho_2, \alpha) / \partial \rho) + H \cdot (T(\rho_2, \alpha) - T_\infty) = 0$ $H=50$ $T_\infty=0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

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Related files: [mad-B.txt](#), [points-4.txt](#), PEEI-mem-4.bin, [cond-3-4.txt](#), [PEEI-sol-3-4.txt](#), [plot-3-4.jpg](#)

Case 3-5:

Conditions: $K \cdot (\partial T(\rho_2, \alpha) / \partial \rho) + H \cdot (T(\rho_2, \alpha) - T_\infty) = 0$ $H=50$ $T_\infty=0$ $\partial T(\rho, \alpha) / \partial \alpha = 0$

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Related files: [mad-B.txt](#), [points-5.txt](#), PEEI-mem-5.bin, [cond-3-5.txt](#), [PEEI-sol-3-5.txt](#), [plot-3-5.jpg](#)

Bibliography:

[1] H. S. CARSLAW, J. C. JAEGER, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1986, London.